Personalized Computational Hemodynamics

Yuri Vassilevski

Marchuk Institute of Numerical Mathematics RAS
Moscow Institute of Physics and Technology
Sechenov University

yuri.vassilevski@gmail.com dodo.inm.ras.ru/vassilevski

Conference “Mathematics in Medicine”

Tomsk

28.05.2021

Co-authors

- S.Simakov
- T.Gamilov
- R.Pryamonosov
- M.Olshanskii
- A.Danilov
- A.Lozovskiy
- V.Salamatova
- A.Liogky

research team based at INM RAS, MIPT, Sechenov University

Co-authors

- S.Simakov
- T.Gamilov
- R.Pryamonosov
- M.Olshanskii
- A.Danilov
- A.Lozovskiy
- V.Salamatova
- A.Liogky

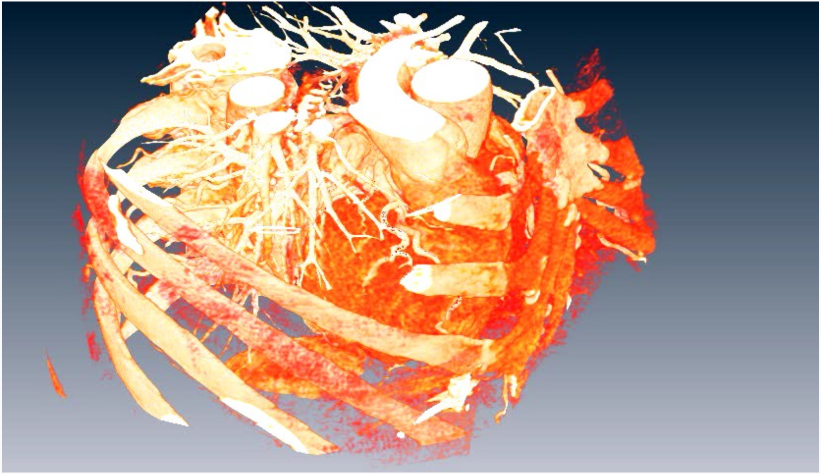
research team based at INM RAS, MIPT, Sechenov University

Financial support

- RSF 2014-2021
- RFBR 2010-2018

Personalized model of coronary hemodynamics

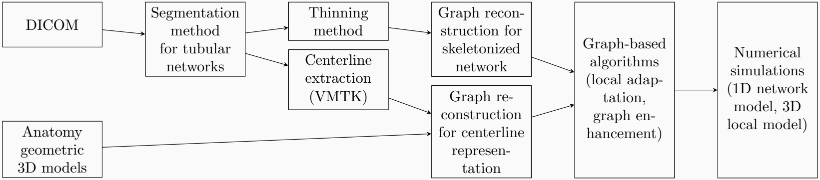
Segmentation of blood vessels

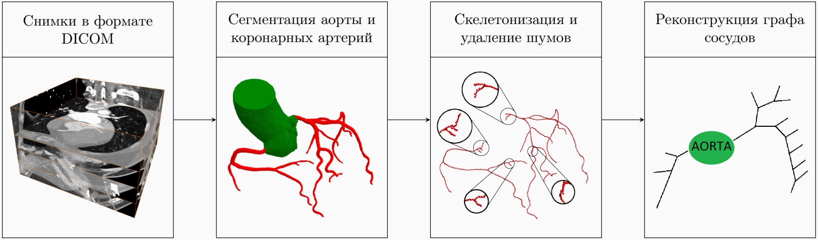


Coronary arteries

Segmentation and skeletonization

R.Pryamonosov, A.Danilov





Technological chain

Danilov A. et al. Methods of graph network reconstruction in personalized medicine. *Int.J.Numer.Meth.Biomed.Engng.*, 2016

Skeletonization efficiency

Skeletons of a coronary tree and of a micro-CT of vascular corrosion cast of rabbit kidney provided by J. Alastruey, Department of Bioengineering, King's College London, UK

	Case 1	Rabbit kidney
Resolution	$512 \times 512 \times 248$	$2000 \times 1989 \times 910$
Distance map	0.20 sec	58.12 sec
Thinning	0.79 sec	526.98 sec
False twigs cleaning	0.15 sec	16.61 sec
Graph construction	0.13 sec	12.27 sec
Skeleton segments	22	4302

A.Kholodov, S.Simakov, A.Favorski, S.Mukhin, N.Sosnin A.Quarteroni,L.Formaggio,E.Toro,...

Mass and momentum balance

$$\partial S_k/\partial t + \partial(S_k u_k)/\partial x = 0,$$
$$\partial u_k/\partial t + \partial\left(u_k^2/2 + p_k/\rho\right)/\partial x = f_{fr}(S_k, u_k),$$

k is index of the tube, t is the time, x is the distance along the tube, ρ is the blood density (constant), $S_k(t,x)$ is the cross-section area, $u_k(t,x)$ is the linear velocity averaged over the cross-section, $p_k(S_k)$ is the blood pressure, f_{fr} is the friction force

S.S.Simakov, A.S.Kholodov. Computational study of oxygen concentration in human blood under low frequency disturbances. *Mathematical Models and Computer Simulations*, 2009 **1** (2)

1D hemodynamic equations

A.Kholodov, S.Simakov, A.Favorski, S.Mukhin, N.Sosnin A.Quarteroni,L.Formaggio,E.Toro,...

At the vessels junctions the Poiseuille's pressure drop and mass conservation

$$p_k\left(S_k\left(t,\tilde{x}_k\right)\right) - p^l_{node}\left(t\right) = \varepsilon_k R^l_k S_k\left(t,\tilde{x}_k\right) u_k\left(t,\tilde{x}_k\right), k = k_1, k_2, \ldots, k_M,$$

$$\sum_{k=k_1,k_2,\ldots,k_M} \varepsilon_k S_k\left(t,\tilde{x}_k\right) u_k\left(t,\tilde{x}_k\right) = 0,$$

$\varepsilon = 1, \tilde{x}_k = L_k$ for incoming tubes, $\varepsilon = -1$, and $\tilde{x}_k = 0$ for outgoing tubes, R^l_k is the hydraulic resistance

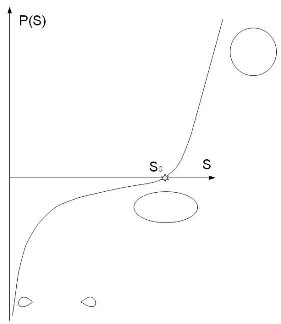
Bessonov N., Sequeira A., Simakov S., Vassilevski Yu., Volpert V. Methods of blood flow modelling. *Math. Model. Nat. Phenom.*, V.11, No.1, 2016

1D hemodynamic equations

A.Kholodov, S.Simakov, A.Favorski, S.Mukhin, N.Sosnin A.Quarteroni,L.Formaggio,E.Toro,...

Elasticity of the tube wall:

$$p_k(S_k) - p_{*k} = \rho c_k^2 f(S_k)$$



Vassilevski Yu., Salamatova V., Simakov S. On the elastisity of blood vessels in one-dimensional problems of hemodynamics.

Computational Mathematics and Mathematical Physics, V.55, No.9, p.1567-1578, 2015.

Coronary hemodynamics

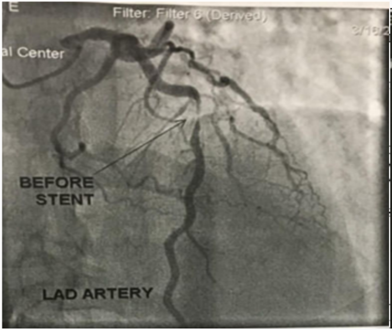
Ischemic heart disease is caused by

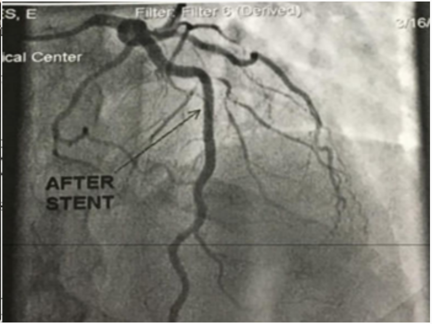
- pathology of microvasculature (therapy)
- pathology of coronary arteries (revascularization)

Coronary hemodynamics

Ischemic heart disease is caused by

- pathology of microvasculature (therapy)
- pathology of coronary arteries (revascularization)

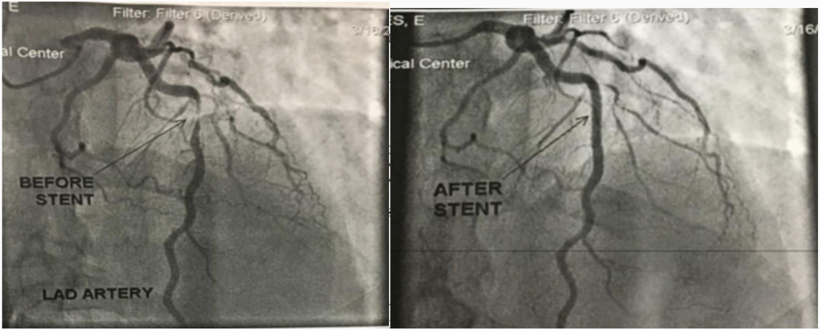




Coronary hemodynamics

Ischemic heart disease is caused by

- pathology of microvasculature (therapy)
- pathology of coronary arteries (revascularization)

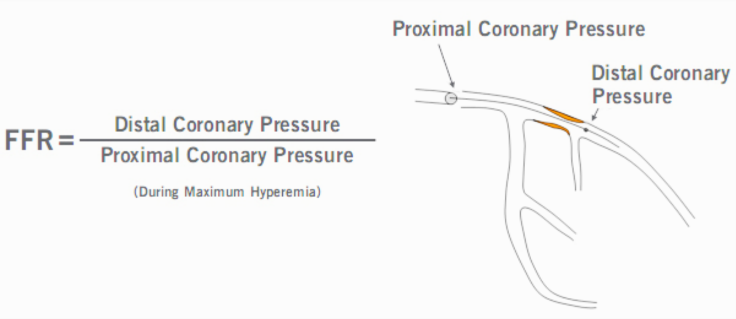


Indication for revascularization

- before 2014: Vascular occlusion factor (relative lesion cross-sectional area) $VOF > 0.5$
- after 2014: Fractional flow reserve $FFR < 0.8$

2014 ESC/EACTS Guidelines on myocardial revascularization. *Eur.Heart J*, 2014 **35** (37)

Fractional flow reserve (FFR)



Clinical practice: endovascular intervention, expensive transducer

Pijls NH, Sels JW., Functional measurement of coronary stenosis. *J.Am. Coll. Cardiol.*, 2012 **59** (12)

Kopylov Ph., Bykova A., Vassilevski Yu., Simakov S. Role of measurement of fractional flow reserve (FFR) in coronary artery atherosclerosis. *Therapeutic archive*, 2015 **87** (9)

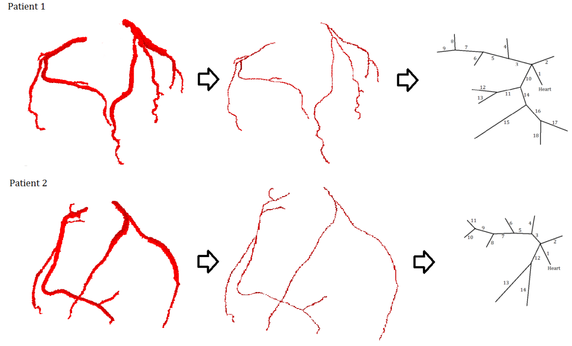
Virtual/Computed fractional flow reserve (vFFR/cFFR)

- Hemodynamic simulation based on personalized data:
  - Computed Tomographic Coronary Angiography (DICOM)
- $FFR = \frac{\overline{P}_{dist}}{\overline{P}_{aortic}}$
- Advantages of vFFR
  - non-invasivity
  - physiological significance of each of multiple lesions
  - virtual stenting
  - applicability to any segment of the coronary tree

Computation of virtual fractional flow reserve

T.Gamilov, S.Simakov

the 1D equations may be adopted to coronary hemodynamics

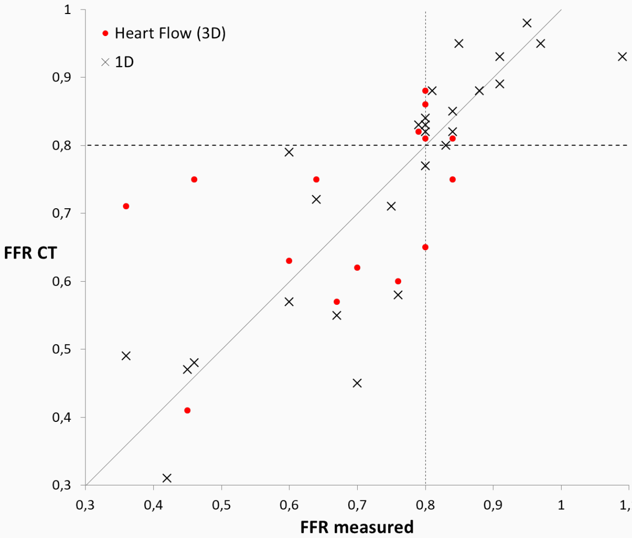


reconstructed arterial part based on two anonymous patient-specific data sets

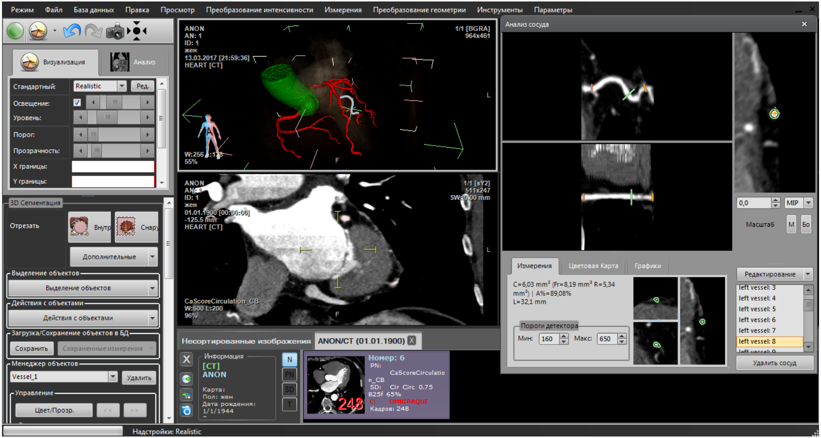
T.Gamilov, Ph.Kopylov, R.Pryamonosov, S.Simakov. Virtual Fractional Flow Reserve Assesment in Patient-Specific Coronary Networks by 1D Hemodynamic Model. *Russ. J. Numer. Anal. Math. Modelling*, 2015 **30** (5)

Computation of virtual fractional flow reserve

T.Gamilov, S.Simakov



FFR_{CT} within Multivox toolbox

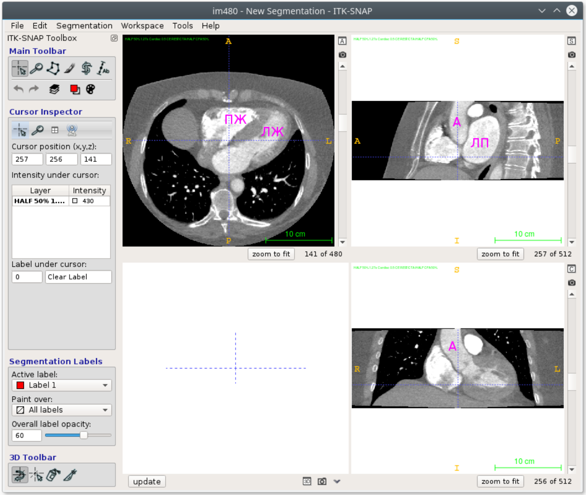


Personalized model of blood flow in human ventricles

Dynamic left ventricle model

- Aim: hemodynamic modeling in heart ventricles
- Dynamics: ventricle walls reconstructed from ceCT images
- Data: ceCT, 100 images, 1.27 seconds
- Resolution: $512 \times 512 \times 480$, raw data – 24 Gb
- Patient: anonymized, female, 50 years old
- Problem: generation of dynamic mesh from ceCT images

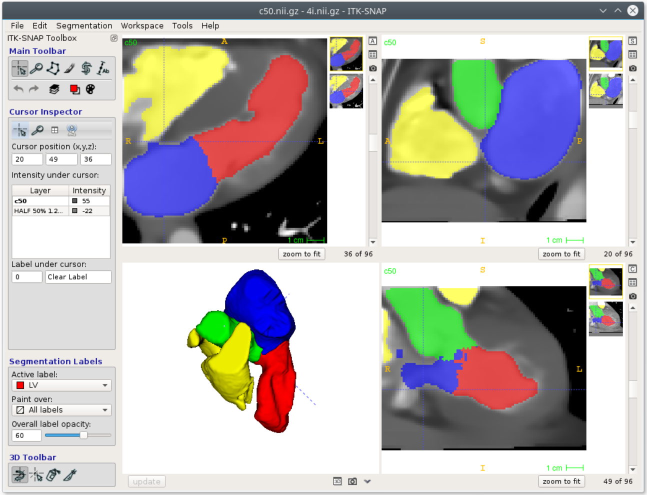
A.Danilov



Initial ceCT image 50

Segmentation of ceCT heart images and ML

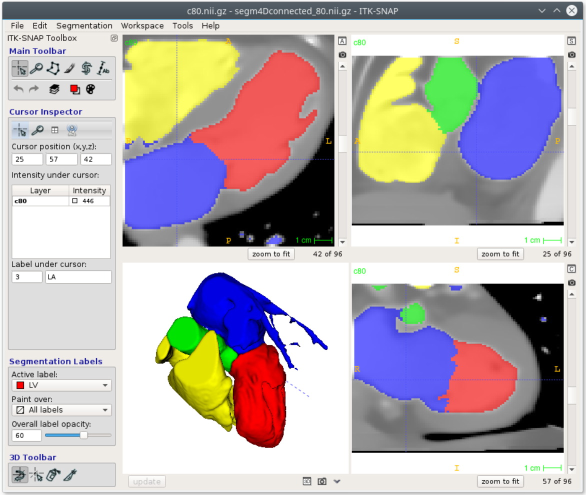
A.Danilov



Manual segmentation 50

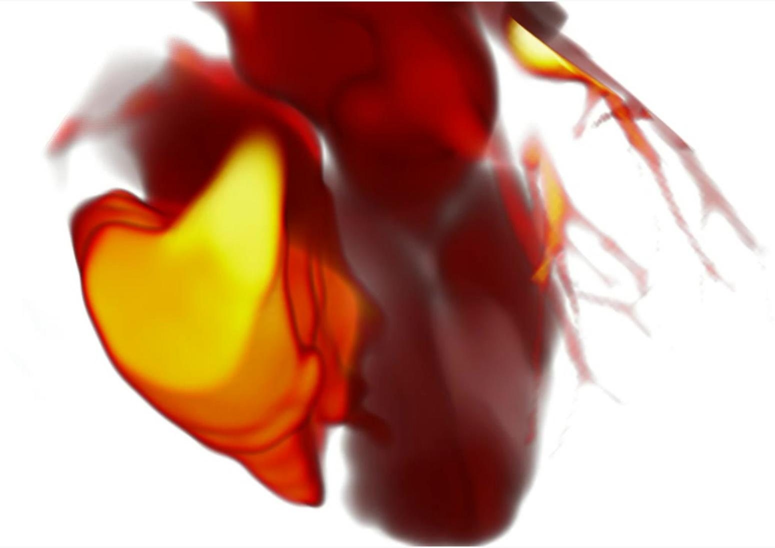
Segmentation of ceCT heart images and ML

A.Danilov



Automatic segmentation 80

Dynamic left ventricle model



Incompressible fluid flow in a moving domain

Let $\boldsymbol{\xi}$ mapping Ω_0 to $\Omega(t)$, $\mathsf{F} = \nabla\boldsymbol{\xi} = \mathsf{I} + \nabla\mathsf{u}$, $J = \det(\mathsf{F})$ be given

Incompressible fluid flow in a moving domain

Let $\boldsymbol{\xi}$ mapping Ω_0 to $\Omega(t)$, $\mathsf{F} = \nabla\boldsymbol{\xi} = \mathsf{I} + \nabla\mathsf{u}$, $J = \det(\mathsf{F})$ be given

Dynamic equations

$$\frac{\partial \mathsf{v}}{\partial t} = (J\rho_f)^{-1}\mathrm{div}\left(J\boldsymbol{\sigma}_f\mathsf{F}^{-T}\right) - \nabla\mathsf{v}\left(\mathsf{F}^{-1}\left(\mathsf{v} - \frac{\partial \mathsf{u}}{\partial t}\right)\right) \quad \text{in } \Omega_0$$

Fluid incompressibility

$$\mathrm{div}\left(J\mathsf{F}^{-1}\mathsf{v}\right) = 0 \quad \text{in } \Omega_0 \quad \text{or} \quad J\nabla\mathsf{v} : \mathsf{F}^{-T} = 0 \quad \text{in } \Omega_0$$

Constitutive relation for the fluid stress tensor

$$\boldsymbol{\sigma}_f = -p_f\mathsf{I} + \mu_f((\nabla\mathsf{v})\mathsf{F}^{-1} + \mathsf{F}^{-T}(\nabla\mathsf{v})^T) \quad \text{in } \Omega_0$$

Incompressible fluid flow in a moving domain

Let $\boldsymbol{\xi}$ mapping Ω_0 to $\Omega(t)$, $\mathsf{F} = \nabla\boldsymbol{\xi} = \mathsf{I} + \nabla\mathsf{u}$, $J = \det(\mathsf{F})$ be given

Dynamic equations

$$\frac{\partial \mathsf{v}}{\partial t} = (J\rho_f)^{-1}\mathrm{div}\,(J\boldsymbol{\sigma}_f\mathsf{F}^{-T}) - \nabla\mathsf{v}\left(\mathsf{F}^{-1}\left(\mathsf{v} - \frac{\partial \mathsf{u}}{\partial t}\right)\right) \quad \text{in } \Omega_0$$

Fluid incompressibility

$$\mathrm{div}\,(J\mathsf{F}^{-1}\mathsf{v}) = 0 \quad \text{in } \Omega_0 \quad \text{or} \quad J\nabla\mathsf{v} : \mathsf{F}^{-T} = 0 \quad \text{in } \Omega_0$$

Constitutive relation for the fluid stress tensor

$$\boldsymbol{\sigma}_f = -p_f\mathsf{I} + \mu_f((\nabla\mathsf{v})\mathsf{F}^{-1} + \mathsf{F}^{-T}(\nabla\mathsf{v})^T) \quad \text{in } \Omega_0$$

Mapping $\boldsymbol{\xi}$ does not define material trajectories $\rightarrow$ quasi-Lagrangian formulation

Finite element scheme

The scheme — Ani3D package, Lipnikov, Vassilevski et al.

- semi-implicit
- produces one linear system per time step
- first order in time (may be generalized to the second order)
- unconditionally stable (no CFL restriction) and 2nd order accurate, proved with assumptions:
  - $\inf_Q J \geq c_J > 0, \quad \sup_Q(\|\mathbf{F}\|_F + \|\mathbf{F}^{-1}\|_F) \leq C_F$
  - LBB-stable pairs (e.g. P_2/P_1)
  - Δt is not large

A.Danilov, A.Lozovskiy, M.Olshanskii, Yu.Vassilevski. A finite element method for the Navier-Stokes equations in moving domain with application to hemodynamics of the left ventricle. *Russian J. Numer. Anal. Math. Modelling*, 32, 2017

A.Lozovskiy, M.Olshanskii, Yu.Vassilevski. A quasi-Lagrangian finite element method for the Navier-Stokes equations in a time-dependent domain. *CMAME*, 333, 2018

3D: left ventricle of a human heart

A.Danilov, A.Lozovskii

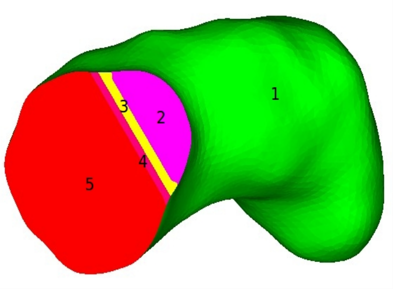


Рис. 1: Left ventricle

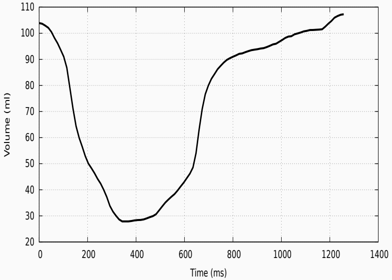
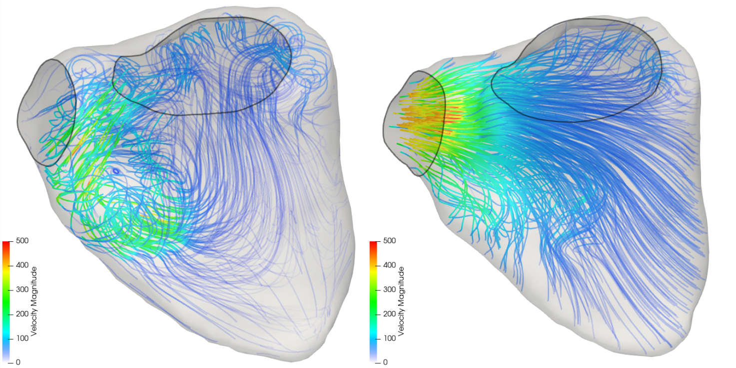


Рис. 2: Ventricle volume

The law of motion for the ventricle walls is known thanks to ceCT scans $\rightarrow$ 100 mesh files with time gap 0.0127 s $\rightarrow$ $\mathbf{u}$ given as input

- 2 - aortic valve (outflow)
- 5 - mitral valve (inflow)

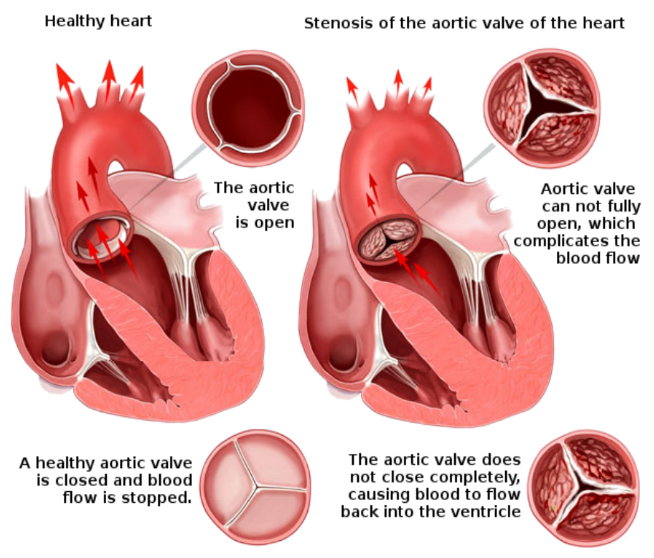
4D CT-based CFD simulation in the RV of a TGA patient



Yu.Vassilevski et al. A stable method for 4D CT-based CFD simulation in the right ventricle of a TGA patient. *Russian J. Numer. Anal. Math. Modelling*, 35, 2020

Personalized model of aortic valve closure

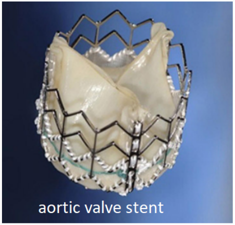
Aortic valve replacement

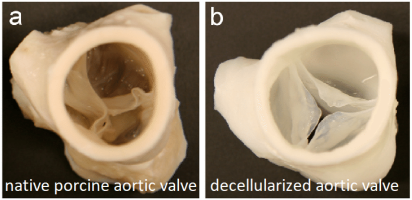


Aortic valve replacement

Surgical treatment of AVD:

- AV replacement using mechanical/biological aortic valve (decellularized aortic homografts)



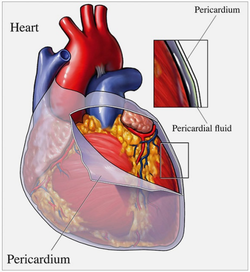


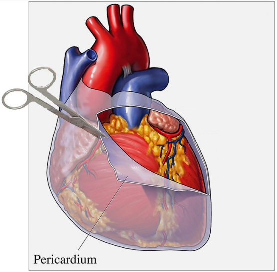
durability; problem of clotting; cost; problem of rejection

- AV cusps replacement by leaflets cut from auto-pericardium
  - no immune response
  - efficient, low-cost
  - all measurements and cuttings are made during operation

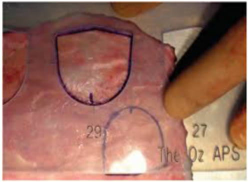
Aortic valve replacement

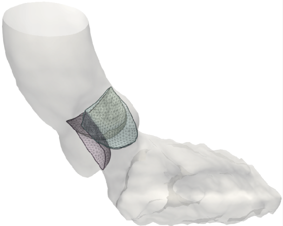
The pericardium is a fluid filled sack that surrounds the heart and the roots of the great vessels.





'Future' leaflets are cut from chemically treated auto-pericardium



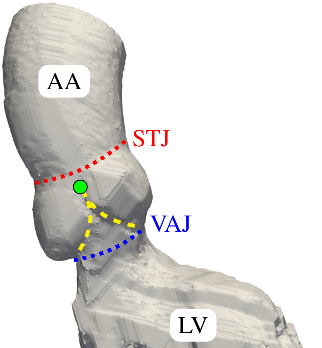


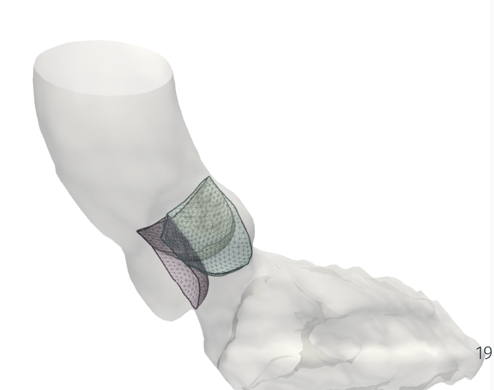
Mathematical modeling of AV replacement

V.Salamatova, A.Liogky

Objectives of modeling:

- degree of regurgitation
- coaptation zone (heights) [demand on computation time for real-time surgical planning system: the results within a few minutes on a personal computer.]

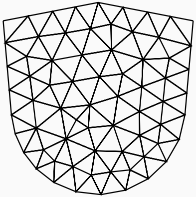




Mathematical modeling of AV replacement

V.Salamatova, A.Liogky

- Finding diastolic state of AV using simplified models



- leaflet is an oriented triangulated surface
- each node has a point mass at which forces due to pressure, elasticity and contacts are applied
- we search static equilibrium
- personalization, real-time simulation, mechanical prop.

$\mathbf{F}_i^e$ elastic force:

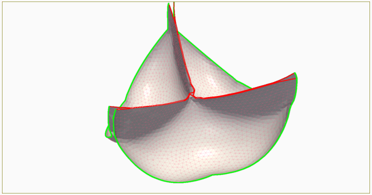
1. Mass-spring model (each edge is a spring with given stiffness)

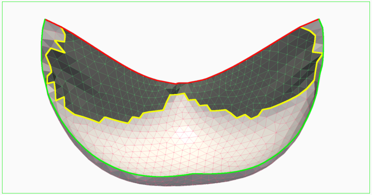
$$\mathbf{F}_i^e = \sum_{e_{ij}} \mathbf{F}_{ij},\ \mathbf{F}_{ij} = k_{ij}(\|\mathbf{r}_j - \mathbf{r}_i\| - L_{ij})\frac{\mathbf{r}_j - \mathbf{r}_i}{\|\mathbf{r}_j - \mathbf{r}_i\|}, \quad k_{ij} = \frac{E(\varepsilon, \alpha_0)HA_{ij}}{L_{ij}^2}$$

2. Hyperelastic nodal force (HNF)

$$\mathbf{F}_i^e = \sum_{T_P \in S_i} \mathbf{F}_i(T_P),\ \mathbf{F}_i(T) = -A_T \frac{\partial U_d(\mathbf{r}_i, \mathbf{r}_j, \mathbf{r}_k)}{\partial \mathbf{r}_i},$$

where discretized counterpart $U_d(\mathbf{r}_i, \mathbf{r}_j, \mathbf{r}_k)$ of elastic potential U is known.

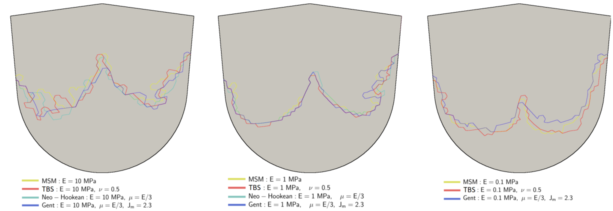


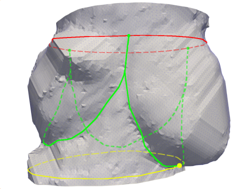


Model	h_E, mm	h_{C-C}, mm	h_{avr}, mm	NCCA, %	CPU time, s
FSI, lin.el.	10.5	1.5	2.7	21	n/a
MSM	10.8	3.8	3.3	25	44
St-V-K (TBS)	10.8	3.1	2.9	24	58
neo-Hookean	10.4	3.0	2.5	21	136
Gent	10.8	3.4	3.1	24	203

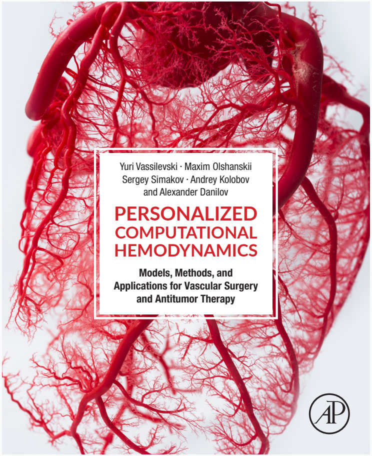
Coaptation profiles for different elastic models

Models and elastic modulus were varied





V.Salamatova, A.Liogky et al. Numerical assessment of coaptation for auto-pericardium based aortic valve cusps. *Russian J. Numer. Anal. Math. Modelling*, 34(5), 2019



Y.Vassilevski, M.Olshanskii, S.Simakov, A.Kolobov, A.Danilov

Personalized Computational Hemodynamics: Models, Methods, and Applications for Vascular Surgery and Antitumor Therapy

Academic Press 2020

Summer school at Sirius

- **What:** Summer school for students
  “Computational technologies, multidimensional data, modelling”
  (numerical algebra, numerical methods, mathematical modelling in Earth and life sciences)

- **When**: August 2 - August 22, 2021

- **Where:** Sochi

- **Organizers:** Sirius University and INM RAS

- **How:** apply before June 2 for full financial support via

  inm.ras.ru or

  sochisirius.ru/obuchenie/graduates/smena901/4333

Где этому научиться?

- Сеченовский Университет, каф.выс.мат, мех. и матмод.
  Механика и математическое моделирование

- МФТИ, школа ПМИ, каф. выч.тех.мод. в геоф. и биом.
  Прикладные математика и физика